Journal of Applied Mechanics and Technical Physics, Vol. 50, No. 6, pp. 989–997, 2009

DIRECT METHOD OF SOLVING THE BOUNDARY-VALUE PROBLEM OF RELAXATION OF RESIDUAL STRESSES IN A HARDENED CYLINDRICAL SPECIMEN UNDER CREEP CONDITIONS

V. P. Radchenko and M. N. Saushkin

UDC 539.376:539.4.014.13

A direct method of solving a boundary-value problem for a surface-hardened cylindrical specimen affected by a tensile load under creep conditions is proposed. Relations for estimating the kinetics of the stress–strain state in the hardened layer are obtained. The adequacy of the solution is verified by experimental data on relaxation of residual stresses in the hardened layer of a cylindrical specimen made of ÉI 691 *steel at* $T = 400°$C*. The calculated and experimental residual stresses are demonstrated to be in good agreement.*

Key words: *cylindrical specimen, plastic surface hardening, residual stresses, creep, relaxation.*

Introduction. When some specimen is in operation, it is its surface layer that is subjected to the most intense mechanical, thermal, and other loads. In most cases, specimen failure (nucleation and propagation of fatigue cracks, corrosion, etc.) begins in the surface layer.

One method for increasing the lifetime of various specimens is inducing compressive residual stresses in the surface layer (hardening). In this case, an increase, for instance, in fatigue strength is mainly caused by compressive residual stresses in the surface layer, which prevent outgoing of various dislocations and vacancies [1, 2]. In the course of specimen operation at high temperatures, however, the creep phenomenon induces relaxation of residual stresses (the absolute value of compressive stresses decreases), which is accompanied by rheological deformation of the structure itself.

An approximate method of solving the problem of relaxation of residual stresses in a hardened cylindrical specimen under tension was proposed in several works [3, 4]. This method is based on cylinder decomposition into a thin hardened layer, which exerts practically no effect on the stiffness of the structure as a whole, and the "body" of the structure. The thin hardened layer 100–200 μm thick is assumed to be "glued" onto the cylinder surface and to be deformed together with the cylinder in the "stiff" loading mode with specified values of strain on the cylinder surface.

The present paper describes a different approach based on the direct solution of the boundary-value problem of creep with a given initial stress–strain state. Implementation of the approach under development includes two stages. During the first stage, the initial (original) stress–strain state formed in the surface layer of the cylinder owing to plastic surface deformation is determined [3–5]. During the second stage, the boundary-value problem of creep of a stretched rod with specified initial fields of residual stresses and plastic strains is solved (in most cases, numerically).

1. Technique of Calculating the Fields of Residual Stresses and Plastic Strains in the Specimen as a Result of Hardening. Let us introduce a standard cylindrical coordinate system (r, θ, z) and indicate the circumferential, radial, and axial components of the residual stress tensor in the cylindrical specimen by $\sigma_\theta^{\rm res}$, $\sigma_r^{\rm res}$,

Samara State Technical University, Samara 443100; radch@samgtu.ru; msaushkin@gmail.com. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, Vol. 50, No. 6, pp. 90–99, November–December, 2009. Original article submitted January 12, 2009.

and $\sigma_z^{\rm res}$, respectively. The plastic strains arising in the cylinder owing to plastic surface strains and initiating residual stresses are denoted by q_θ, q_r, and q_z.

The shear components of the residual stress and strain tensors are assumed to be small, which is valid for a number of technological modes of hardening (hydraulic shot blasting, bombardment by microspheres, etc.), and can be neglected. The normal components of plastic strains induced by hardening are described by the equation

$$q_\theta(r) = q_z(r) \tag{1}$$

corresponding to the hypothesis that the strains on an infinitely small area of the surface layer of the cylindrical specimen are induced in the same manner as in the half-space layer. In addition, secondary plastic strains in the compressed layer are assumed to be absent.

We assume that the circumferential component $\sigma_\theta^{\rm res}(r)$ is determined in experiments; therefore, the problem posed reduces to calculating the stresses $\sigma_r^{\rm res}(r)$ and $\sigma_z^{\rm res}(r)$, and also the residual plastic strains from the measured values of the component $\sigma_\theta^{\rm res}(r)$ [3–5].

The equilibrium equation

$$r\,\frac{d\sigma_r^{\rm res}}{dr} + \sigma_r^{\rm res} = \sigma_\theta^{\rm res} \tag{2}$$

implies that the stress diagram $\sigma_\theta^{\rm res}(r)$ should be self-balanced:

$$\int\limits_0^a \sigma_\theta^{\rm res}(r)\,dr = \int\limits_0^a \frac{d}{dr}\,(r\sigma_r^{\rm res}(r)) = 0 \tag{3}$$

(a is the radius of the cylindrical specimen). In deriving Eq. (3), we used the condition $\sigma_r^{\rm res}(a) = 0$, which means that the cylindrical specimen after hardening is in its natural unloaded state.

From the equilibrium equation (2), we obtain the equality

$$\sigma_r^{\rm res}(r) = \frac{1}{r}\int\limits_0^r \sigma_\theta^{\rm res}(z)\,dz, \tag{4}$$

which allows $\sigma_r^{\rm res}(r)$ to be calculated from the measured values of the function $\sigma_\theta^{\rm res}(r)$. It can be easily demonstrated that

$$\lim_{r\to 0} \sigma_r^{\rm res}(r) = \sigma_\theta^{\rm res}(0) \overset{\rm def}{=} \sigma_0. \tag{5}$$

It follows from Eqs. (2)–(5) that the stress diagrams $\sigma_\theta^{\rm res}(r)$ and $\sigma_r^{\rm res}(r)$ are similar to the diagrams plotted in Fig. 1.

The distribution of the component $\sigma_z^{\rm res}(r)$ can be calculated only with allowance for residual plastic strains.

The principal components of the tensor of the total strain of the cylindrical specimen ε_i^0 ($i \equiv r,\ \theta,\ z$), which is acquired as a result of hardening, can be presented as

$$\varepsilon_i^0(r) = e_i^0(r) + q_i(r), \tag{6}$$

where e_i^0 and q_i are the components of the tensors of elastic and plastic strains, respectively.

From the condition of incompressibility under plastic strains $q_z + q_\theta + q_r = 0$, Eqs. (1) and (6), and the equation of compatibility of strains

$$r\,\frac{d\varepsilon_\theta^0}{dr} + \varepsilon_\theta^0 = \varepsilon_r^0, \tag{7}$$

we obtain the equality

$$q_\theta = q_z = -q_r/2 \tag{8}$$

and the differential equation for the circumferential component q_θ:

$$r\,\frac{dq_\theta}{dr} + 3q_\theta = e_r^0 - r\,\frac{de_\theta^0}{dr} - e_\theta^0. \tag{9}$$

The elastic strains included into Eq. (9) can be easily expressed via the residual stresses from Hooke's law:

$$e_r^0 = [\sigma_r^{\rm res} - \mu(\sigma_\theta^{\rm res} + \sigma_z^{\rm res})]/E, \qquad e_\theta^0 = [\sigma_\theta^{\rm res} - \mu(\sigma_r^{\rm res} + \sigma_z^{res})]/E \tag{10}$$

(μ is Poisson's ratio and E is Young's modulus).

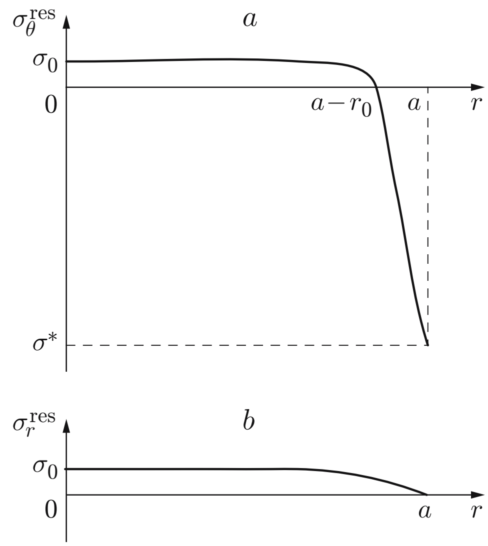

Fig. 1. Diagrams $\sigma_\theta^{\rm res}(r)$ (a) and $\sigma_r^{\rm res}(r)$ (b) after hardening of the cylindrical specimen.

In addition to the known components $\sigma_\theta^{\rm res}$ and $\sigma_r^{\rm res}$, Eq. (10) includes an unknown component $\sigma_z^{\rm res}$. To determine $\sigma_z^{\rm res}$, we have to introduce an additional assumption, in particular, a hypothesis of plane sections for the cylindrical specimen. In other words, the cross sections of the cylindrical specimen, which are plane before the beginning of hardening, are assumed to remain plane after the hardening process, and

$$e_z^0(r) + q_z(r) = \varepsilon_z^* \qquad (\varepsilon_z^* = \text{const}, \quad r \in [0; a]). \tag{11}$$

Constraint (11) can be violated only near the free end faces of the cylinder.

Expressing e_z^0 in terms of stresses [similar to Eq. (10)] and applying appropriate transformations, we obtain the following relations from Eq. (11):

$$\sigma_z^{\rm res}(r) = E(\varepsilon_z^* - q_z(r)) + \mu(\sigma_r^{\rm res}(r) + \sigma_\theta^{\rm res}(r)). \tag{12}$$

Using Eq. (12), we can eliminate $\sigma_z^{\rm res}$ from Eqs. (10):

$$\begin{aligned} e_r^0 &= (1+\mu)[(1-\mu)\sigma_r^{\rm res} - \mu\sigma_\theta^{\rm res}]/E - \mu(\varepsilon_z^* - q_z), \\ e_\theta^0 &= (1+\mu)[(1-\mu)\sigma_\theta^{\rm res} - \mu\sigma_r^{\rm res}]/E - \mu(\varepsilon_z^* - q_z). \end{aligned} \tag{13}$$

Taking into account Eqs. (8) and (13), we can write Eq. (9) in the form

$$r\,\frac{dq_\theta(r)}{dr} + \frac{3}{1+\mu}\,q_\theta(r) = g(r), \tag{14}$$

where

$$g(r) = \frac{\sigma_r^{\rm res}(r) - \sigma_\theta^{\rm res}(r)}{E} - \frac{r}{E}\Big((1-\mu)\,\frac{d\sigma_\theta^{\rm res}(r)}{dr} - \mu\,\frac{d\sigma_r^{\rm res}(r)}{dr}\Big). \tag{15}$$

The general solution of the differential equation (14) has the form

$$q_\theta(r) = \frac{1}{r^{3/(1+\mu)}}\Big(\int\limits_0^r z^{(2-\mu)/(1+\mu)} g(z)\,dz + C\Big). \tag{16}$$

As plastic strains arise only within a moderate depth in the case of surface hardening, we can use the following hypothesis: $\lim\limits_{r\to 0} q_\theta(r) = 0$. It can be easily shown that $C = 0$ in Eq. (16) in this case. Then, substituting Eq. (15) into Eq. (16), we obtain

$$q_\theta(r) = \frac{1-2\mu}{E(1+\mu)r^{3/(1+\mu)}}\int\limits_0^r z^{(2-\mu)/(1+\mu)}[\sigma_r^{\rm res}(z) + 2\sigma_\theta^{\rm res}(z)]\,dz - \frac{1}{E}\,[(1-\mu)\sigma_\theta^{\rm res}(r) - \mu\sigma_r^{\rm res}(r)]. \tag{17}$$

The fields of residual plastic strains can now be reconstructed completely: q_θ is calculated by Eq. (17), and then q_r and q_z are calculated by Eq. (8).

According to Eq. (12), to determine the component $\sigma_z^{\rm res}(r)$, it suffices to find ε_z^* by using the constraint of the zero total axial force acting on the specimen:

$$\int\limits_0^a r\sigma_z^{\rm res}(r)\,dr = 0.$$

From the last constraint and Eq. (12), we find the expression

$$\varepsilon_z^* = \frac{2}{a^2}\int\limits_0^a r\Big(q_z(r) - \frac{\mu}{E}\,[\sigma_r^{\rm res}(r) + \sigma_\theta^{\rm res}(r)]\Big)\,dr. \tag{18}$$

Calculating ε_z^* in accordance with Eq. (18), we can apply Eq. (12) to determine uniquely the distribution of the component $\sigma_z^{\rm res}(r)$.

Thus, the global scheme for calculating the fields of residual stresses and plastic strains in the hardened layer of the cylindrical specimen has the form

$$\sigma_\theta^{\rm res}(r) \xrightarrow{(4)} \sigma_r^{\rm res}(r) \xrightarrow{(17)} q_\theta(r) \xrightarrow{(8)} q_r(r), q_z(r) \xrightarrow{(18)} \varepsilon_z^* \xrightarrow{(12)} \sigma_z^{\rm res}(r). \tag{19}$$

The arrows here indicate the sequence of determining these quantities; the numbers above the arrows are the numbers of equations used to determine these quantities.

2. Solution of the Boundary-Value Problem of Relaxation of Residual Stresses in the Hardened Layer of the Cylindrical Specimen Under Creep Conditions. Let us consider a cylindrical specimen of radius $r = a$ with fields of residual stresses and plastic strains induced in the surface layer, which is subjected to the action of a tensile longitudinal force $F(t)$.

The formulation of the boundary-value problem at an arbitrary time t includes the following equations:

— equations of equilibrium

$$r\,\frac{d\sigma_r(r,t)}{dr} + \sigma_r(r,t) = \sigma_\theta(r,t); \tag{20}$$

$$\int\limits_0^a \sigma_z(r,t)r\,dr = \frac{F(t)}{2\pi}, \tag{21}$$

where $\sigma_r(r,t)$, $\sigma_\theta(r,t)$, and $\sigma_z(r,t)$ are the radial, circumferential, and axial components of the stress tensor in the cylinder, respectively;

— equation of compatibility of strains

$$r\,\frac{d\varepsilon_\theta(r,t)}{dr} + \varepsilon_\theta(r,t) = \varepsilon_r(r,t), \tag{22}$$

where $\varepsilon_r(r,t)$ and $\varepsilon_\theta(r,t)$ are the radial and circumferential components of the total strain tensor;

— hypothesis of plane sections

$$\varepsilon_z(r,t) = \varepsilon_z^*(t), \tag{23}$$

where $\varepsilon_z(r,t)$ is the axial component of the total strain tensor;

— boundary constraints

$$\sigma_r(r,t)\Big|_{r=0} = 0.$$

Let us formulate the initial constraints. Immediately after hardening (at the time $t = 0-0$), the stress–strain state of the cylinder is described by the stresses $\sigma_i^{\rm res}(r)$ $(i \equiv r, \theta, z)$ and by the relations for strains, which follow from Eq. (6) and Hooke's law:

$$\varepsilon_r^0(r) = [\sigma_r^{\rm res}(r) - \mu(\sigma_\theta^{\rm res}(r) + \sigma_z^{\rm res}(r))]/E + q_r(r),$$

$$\varepsilon_\theta^0(r) = [\sigma_\theta^{\rm res}(r) - \mu(\sigma_r^{\rm res}(r) + \sigma_z^{\rm res}(r))]/E + q_\theta(r), \qquad \varepsilon_z^0(r) = [\sigma_z^{\rm res}(r) - \mu(\sigma_r^{\rm res}(r) + \sigma_\theta^{\rm res}(r))]/E + q_z(r).$$

Let a longitudinal tensile force $F_0 = \sigma_{z0}\pi a^2$ (σ_{z0} is the axial stress) be applied to the cylinder at the time $t = 0 + 0$. In this case, there occurs an "elastic" jump of the axial stresses

$$\sigma_z(r, 0+0) = \sigma_z^{\text{res}}(r) + \sigma_{z0} \tag{24}$$

and, as a consequence, a jump of the strains

$$\begin{aligned}
\varepsilon_r(r, 0+0) &= [\sigma_r^{\text{res}}(r) - \mu(\sigma_\theta^{\text{res}}(r) + \sigma_z(r, 0+0))]/E + q_r(r), \\
\varepsilon_\theta(r, 0+0) &= [\sigma_\theta^{\text{res}}(r) - \mu(\sigma_r^{\text{res}}(r) + \sigma_z(r, 0+0))]/E + q_\theta(r), \\
\varepsilon_z(r, 0+0) &= [\sigma_z(r, 0+0) - \mu(\sigma_r^{\text{res}}(r) + \sigma_\theta^{\text{res}}(r))]/E + q_z(r).
\end{aligned} \tag{25}$$

Equations (24) and (25), which define the initial stress–strain state of the cylinder after plastic surface hardening and loading of the cylinder by the longitudinal tensile force, are the initial data for the boundary-value problem.

Equations (20)–(25) are closed by the constitutive relations in the differential form, which relate the components of the creep strain and stress tensors (the load F_0 is such that no additional plastic strains appear in the cylinder cross section).

The components of the total strain tensor in the cylinder with induced fields of plastic strains at an arbitrary time t are presented as the sum

$$\varepsilon_i(r, t) = e_i(r, t) + q_i(r) + p_i(r, t), \qquad i \equiv r, \theta, z, \tag{26}$$

where $p_i(r, t)$ is the creep strain, which is calculated by any creep theory that ensures an adequate description of the corresponding experimental data.

At high temperatures and loads, redistribution (relaxation) of the induced residual stresses occurs in the hardened cylindrical specimen owing to the creep strain. To describe the relaxation process, system (20)–(26) should be resolved with respect to the stresses $\sigma_i(r, t)$ ($i \equiv r, \theta, z$), which is the goal of our further study.

For the axial component ε_z, Eqs. (23) and (26) yield

$$e_z(r, t) + q_z(r) + p_r(r, t) = \varepsilon_z^*(t). \tag{27}$$

Hooke's law for elastic strains is written as

$$e_r(r, t) = [\sigma_r(r, t) - \mu(\sigma_\theta(r, t) + \sigma_z(r, t))]/E; \tag{28}$$

$$e_\theta(r, t) = [\sigma_\theta(r, t) - \mu(\sigma_r(r, t) + \sigma_z(r, t))]/E; \tag{29}$$

$$e_z(r, t) = [\sigma_z(r, t) - \mu(\sigma_r(r, t) + \sigma_\theta(r, t))]/E. \tag{30}$$

Taking into account Eq. (30), we find the following relation from Eq. (27):

$$[\sigma_z(r, t) - \mu(\sigma_r(r, t) + \sigma_\theta(r, t))]/E + q_z(r) + p_z(r, t) = \varepsilon_z^*(t),$$

whence it follows that

$$\sigma_z(r, t) = [\varepsilon_z^*(t) - q_z(r) - p_z(r, t)]E + \mu(\sigma_r(r, t) + \sigma_\theta(r, t)). \tag{31}$$

Subtracting Eq. (29) from Eq. (28), we eliminate the component σ_z:

$$e_r(r, t) - e_\theta(r, t) = (1+\mu)[\sigma_r(r, t) - \sigma_\theta(r, t)]/E. \tag{32}$$

With allowance for Eq. (20), Eq. (32) acquires the form

$$e_r(r, t) - e_\theta(r, t) = -\frac{1+\mu}{E}\Big(r\,\frac{d\sigma_r(r, t)}{dr}\Big). \tag{33}$$

We differentiate Eq. (29) with respect to r:

$$\frac{de_\theta(r, t)}{dr} = \frac{1}{E}\Big[\frac{d\sigma_\theta(r, t)}{dr} - \mu\Big(\frac{d\sigma_r(r, t)}{dr} + \frac{d\sigma_z(r, t)}{dr}\Big)\Big]. \tag{34}$$

Hereinafter, the variable t is a parameter; hence, the transformations involve the operator of the total derivative with respect to the variable r.

Differentiating Eq. (31) with respect to the variable r, taking into account the condition $d\varepsilon_z^*(t)/dr = 0$, and substituting the resultant relation into Eq. (34), we obtain

$$\frac{de_\theta(r,t)}{dr} = \frac{1+\mu}{E}\Big[(1-\mu)\,\frac{d\sigma_\theta(r,t)}{dr} - \mu\,\frac{d\sigma_r(r,t)}{dr} + \frac{\mu E}{1+\mu}\Big(\frac{dq_z(r)}{dr} + \frac{dp_z(r,t)}{dr}\Big)\Big]. \tag{35}$$

It follows from Eq. (20) that

$$\frac{d\sigma_\theta(r,t)}{dr} = 2\,\frac{d\sigma_r(r,t)}{dr} + r\,\frac{d^2\sigma_r(r,t)}{dr^2}. \tag{36}$$

Using Eq. (36), we eliminate $d\sigma_\theta/dr$ from Eq. (35):

$$\frac{de_\theta(r,t)}{dr} = \frac{1+\mu}{E}\Big[r(1-\mu)\,\frac{d^2\sigma_r(r,t)}{dr^2} + (2-3\mu)\,\frac{d\sigma_r(r,t)}{dr} + \frac{\mu E}{1+\mu}\Big(\frac{dq_z(r)}{dr} + \frac{dp_z(r,t)}{dr}\Big)\Big]. \tag{37}$$

Taking into account Eqs. (26) and (33), we transform Eq. (22) to the following form:

$$r\,\frac{de_\theta(r,t)}{dr} = -\frac{1+\mu}{E}\Big(r\,\frac{d\sigma_r(r,t)}{dr}\Big) + (q_r(r) - q_\theta(r)) + (p_r(r,t) - p_\theta(r,t)) - r\Big(\frac{dq_\theta(r)}{dr} + \frac{dp_\theta(r,t)}{dr}\Big). \tag{38}$$

Substituting Eq. (37) into Eq. (38) and taking into account Eq. (8), we obtain the ordinary differential equation with respect to σ_r

$$r^2\,\frac{d^2\sigma_r(r,t)}{dr^2} + 3r\,\frac{d\sigma_r(r,t)}{dr} = g(r,t) \tag{39}$$

with the boundary constraints

$$\sigma_r(r,t)\Big|_{r=a} = 0, \qquad \lim_{r\to 0}\frac{d\sigma_r(r,t)}{dr} = 0. \tag{40}$$

Here, we have

$$g(r,t) = \frac{E}{1-\mu^2}\Big[\frac{3}{2}\,q_r(r) + p_r(r,t) - p_\theta(r,t) - r\Big(\frac{dp_\theta(r,t)}{dr} + \mu\,\frac{dp_z(r,t)}{dr}\Big) + \frac{r}{2}\,(1+\mu)\,\frac{dq_r(r)}{dr}\Big].$$

With allowance for Eq. (40), Eq. (39) is written as

$$\sigma_r(r,t) = -\int_r^a \frac{1}{\xi^3}\,\Big(\int_0^\xi g(\eta,t)\eta\,d\eta\Big)\,d\xi. \tag{41}$$

Knowing σ_r, we find σ_θ from Eq. (20):

$$\sigma_\theta(r,t) = \sigma_r(r,t) + r\,\frac{d\sigma_r(r,t)}{dr}. \tag{42}$$

To determine $\sigma_z(r,t)$ by Eq. (31), we have to know the value of $\varepsilon_z^*(t)$. Substituting Eq. (31) into Eq. (21), we obtain the equation with respect to $\varepsilon_z^*(t)$, which implies that

$$\varepsilon_z^*(t) = \frac{1}{E}\,\sigma_{z0} + \frac{2}{a^2}\int_0^a \Big(q_z(r) + p_z(r,t) - \frac{\mu}{E}\,(\sigma_r(r,t) + \sigma_\theta(r,t))\Big)r\,dr.$$

Knowing $\varepsilon_z^*(t)$, we find σ_z from Eq. (31):

$$\sigma_z(r,t) = [\varepsilon_z^* - q_z(r) - p_z(r,t)]E + \mu[\sigma_r(r,t) + \sigma_\theta(r,t)]. \tag{43}$$

Thus, Eqs. (41)–(43) allow us to determine the kinetics of the stresses in the cylindrical specimen under creep conditions; it should be noted that $\sigma_i(r,0) = \sigma_i^{\rm res}(r)$ because $p_i(r,0) = 0$ ($i \equiv r,\theta,z$).

At $t > 0$, the creep strains $p_i(r,t)$ ($i \equiv r,\theta,z$) are calculated from the stresses $\sigma_i(r,t)$ in accordance with the chosen creep theory.

3. Calculation of Relaxation of Residual Stresses in the Cylindrical Specimen and Verification of the Adequacy of the Method to Experimental Data. Numerical implementation of the proposed method and verification of its adequacy to experimental results were performed with the data [1] for the cylindrical specimen of radius $a = 3.75$ mm made of ÉI 691 steel, which was hardened by diamond burnishing and then subjected to thermal exposure (high temperature without loading) during 100 h at a temperature $T = 400$°C.

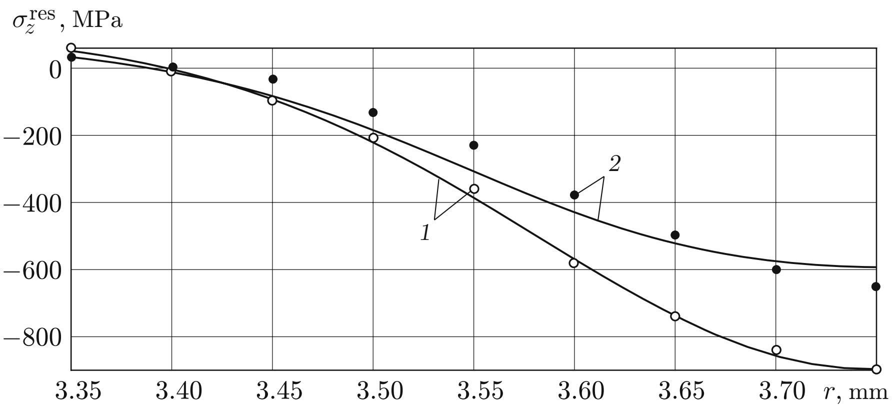

Fig. 2. Experimental diagrams (points) and calculated diagrams (solid curves) of residual stresses in the cylindrical specimen after thermal exposure: $t = 0 - 0$ (1) and $t = 100$ h (2).

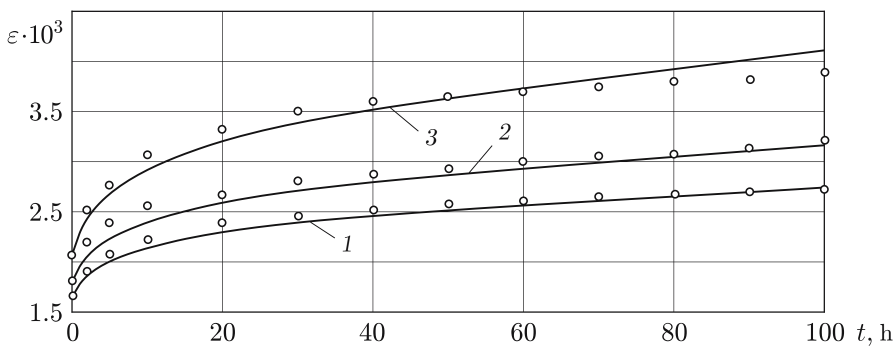

Fig. 3. Experimental dependences (points) and calculated dependences (solid curves) of inelastic rheological deformation of the cylindrical specimen made of ÉI 691 steel versus time at $T = 500^\circ$C: $\sigma = 240$ (1) 260 (2), and 300 MPa (3).

Figure 2 shows the experimental data [1] for the axial component of residual stresses $\sigma_z^{\rm res}(r)$ in the specimen after its hardening and after the end of the creep process ($t = 100$ h) under the action of self-balanced residual stresses induced by thermal exposure. The calculated diagram of the axial component is also plotted in the figure.

According to the method described in Sec. 1, the initial information for calculating the fields of residual stresses is the circular component $\sigma_\theta^{\rm res}(r)$, for which the following approximation can be used (see Fig. 1) [3, 4]:

$$\sigma_\theta^{\rm res}(r) = \sigma_0 + \sigma_1 \exp\left(-(a-r)^2/b^2\right)$$

(σ_0, σ_1, and b are the approximation parameters).

As the experimental data for the component $\sigma_\theta^{\rm res}(r)$ are unavailable, the parameters σ_0, σ_1, and b were varied. For each set of these parameters, a numerical calculation by Eq. (19) was performed until the functional of the root-mean-square deviation of the calculated values of $\sigma_z^{\rm res}(r)$ from the experimental data was minimized. As a result, the following values of the parameters were obtained: $\sigma_0 = 57.47$ MPa, $\sigma_1 = 1033.47$ MPa, and $b = 0.235$ mm.

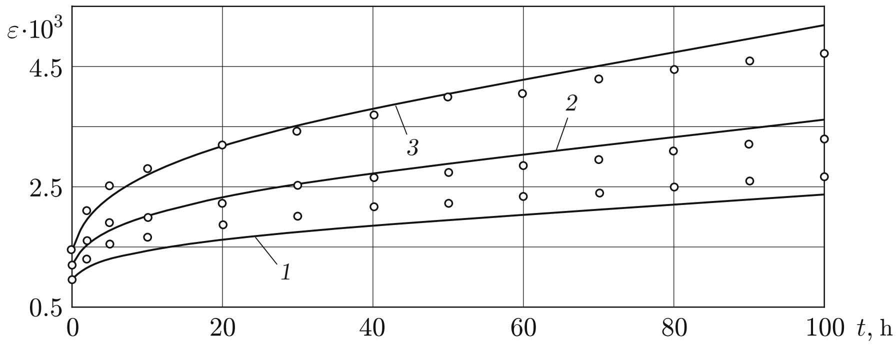

Fig. 4. Experimental dependences (points) and calculated dependence (solid curves) of inelastic rheological deformation of the cylindrical specimen made of ÉI 691 steel versus time at $T = 550°$C: $\sigma = 120$ (1), 150 (2), and 180 MPa (3).

To calculate the relaxation of residual stresses in the course of thermal exposure at $T = 400°$C, experimental data on creep at this temperature should be available. Publications, however, give such data only for $T = 500$ and 550°C (Figs. 3 and 4). The values of Young's moduli for ÉI 691 steel at these temperatures are $E = 142{,}100$ and 122,500 MPa, respectively. As the creep curves plotted in Figs. 3 and 4 do not include the third stage of creep, the model used for calculations was a rheological model [6] of the form

$$p_{ij}(t) = v_{ij}(t) + w_{ij}(t), \qquad \dot{w}_{ij} = (3/2)cS^{m-1}(\sigma_{ij} - \sigma_I\delta_{ij}/3); \tag{44}$$

$$v_{\omega\omega}(t) = \sum_{k=1}^{N} v_{\omega\omega}^k(t);$$

$$v_{\omega\omega}^k(t) = (1+\mu')\beta_{\omega\omega}^k(t) - \mu'(\beta_{11}^k(t) + \beta_{22}^k(t) + \beta_{33}^k(t)); \tag{45}$$

$$\dot{\beta}_{\omega\omega}^k = \begin{cases} \lambda_k\Lambda_k, & \Lambda_k\sigma_{\omega\omega} > 0, \\ 0, & \Lambda_k\sigma_{\omega\omega} \leqslant 0, \end{cases}$$

where $\Lambda_k = b_kS^{n-1}\sigma_{\omega\omega} - \beta_{\omega\omega}^k$, w_{ij} and v_{ij} are the viscous and viscoplastic components of the creep strain p_{ij}, λ_k, c, b_k, n, and N are the model constants used to describe the first and second stages of creep, μ' is Poisson's ratio for the viscoplastic component; $\sigma_I = \sigma_{11} + \sigma_{22} + \sigma_{33}$, S is the stress intensity; the component v_{ij} is calculated in the principal axes.

In the uniaxial case with $\sigma = \text{const}$, the solution of Eqs. (44), (45) yields the following dependence:

$$p(t) = \sum_{k=1}^{N} b_k(1 - \mathrm{e}^{-\lambda_k t})\sigma^m + c\sigma^n t. \tag{46}$$

Using Eq. (46) and the method proposed in [7], we obtained the approximations of the creep curves at $T = 500$ and 550°C, and then the temperature approximation in the form

$$p(t,\sigma,T) = [0.9\cdot 10^{-12}(1-\mathrm{e}^{-0.6t}) + 2.1\cdot 10^{-12}(1-\mathrm{e}^{-0.08t})]\,\mathrm{e}^{0.0242T}\,\sigma^{2.37} + 5.55\cdot 10^{-34}\,\mathrm{e}^{0.107T}\,\sigma^{(644-0.95T)/50}t \tag{47}$$

($T = 500$–550°C). The solid curves in Figs. 3 and 4 are the creep curves calculated by Eq. (47).

Using extrapolation, we obtain the following dependence for $T = 400°$C from Eq. (47):

$$p_z(t,\sigma) = [1.44\cdot 10^{-8}(1-\mathrm{e}^{-0.6t}) + 3.36\cdot 10^{-8}(1-\mathrm{e}^{-0.08t})]\sigma^{2.37} + 2.148\cdot 10^{-15}\sigma^{5.28}t. \tag{48}$$

Thus, Eqs. (48) make it possible to determine all parameters of model (44), (45) at $T = 400°$C: $\lambda_1 = 0.6$, $\lambda_2 = 0.08$, $m = 2.37$, $b_1 = 1.44\cdot 10^{-8}$, $b_2 = 3.36\cdot 10^{-8}$, $c = 2.148\cdot 10^{-15}$, and $n = 5.28$; the value of μ' was assumed to be 0.42. In addition, $E = 161{,}700$ MPa and $\mu = 0.3$.

Following model (44), (45), we implemented the method of calculating the creep process in the cylindrical specimen with self-balanced fields of residual stresses at $T = 400°$C. The problem was solved numerically. Discretization of the domain $0 \leqslant r \leqslant a$ was used, and the time interval with the creep strain was divided by the points $t_0 = 0, t_1, t_2, \ldots$ into the intervals $[t_j, t_{j+1}]$. The system of differential equations (44) and (45) was solved numerically in each interval (e.g., by the Euler method). In the course of the solution, all integrals were calculated by appropriate quadrature formulas, and the derivatives were approximated by finite-different relations.

The kinetics of the stress $\sigma_z(r, t)$ at the final time $t = 100$ h for ÉI-691 steel is illustrated in Fig. 2, which shows good agreement between the calculated and experimental data.

This work was supported by the Russian Foundation for Basic Research (Grant No. 07-01-00478-a) and by the Federal Agency of Education (Grant No. RNP 2.1.1/3397).

REFERENCES

1. V. F. Pavlov, V. A. Kirpichev, and V. B. Ivanov, *Residual Stresses and Fatigue Strength of Hardened Specimens with Stress Concentrators* [in Russian], Samara Research Center, Russian Acad. of Sci., Samara (2008).
2. V. F. Pavlov, "Effect of the character of the distribution of residual stresses across the surface layer of the specimen on the fatigue strength," *Izv. Vyssh. Ucheb. Zaved., Mashinostr.*, No. 7, 3–6 (1987).
3. V. P. Radchenko and M. N. Saushkin, "Mathematical models of recovery and relaxation of residual stresses in a surface-hardened layer of cylindrical specimens under creep conditions," *Izv. Vyssh. Ucheb. Zaved., Mashinostr.*, No. 11, 3–17 (2004).
4. V. P. Radchenko and M. N. Saushkin, "Calculation of relaxation of residual stresses in a surface-hardened layer of a cylindrical specimen under creep conditions," *Vestn. Sam. Gos. Tekh. Univ., Ser. Fiz.-Mat. Nauki*, No. 12, 61–73 (2001).
5. V. P. Radchenko and M. N. Saushkin, *Creep and Relaxation of Residual Stresses in Hardened Structures* [in Russian], Mashinostroenie-1, Moscow (2005).
6. V. P. Radchenko and Yu. A. Eremin, *Rheological Deformation and Failure of Materials and Structural Elements* [in Russian], Mashinostroenie-1, Moscow (2004).
7. Yu. P. Samarin, *Equations of State of Materials with Complex Rheological Properties* [in Russian], Kuibyshev State University, Kuibyshev (1979).